

# Environmental Stochasticity and Outbreaks in Population Dynamics

Authors:

Gabriele Vissio (speaker)

Antonello Provenzale



Consiglio Nazionale delle Ricerche



# The methodology

$$x_{n+1} = r x_n \left(1 - \frac{x_n}{k}\right)$$

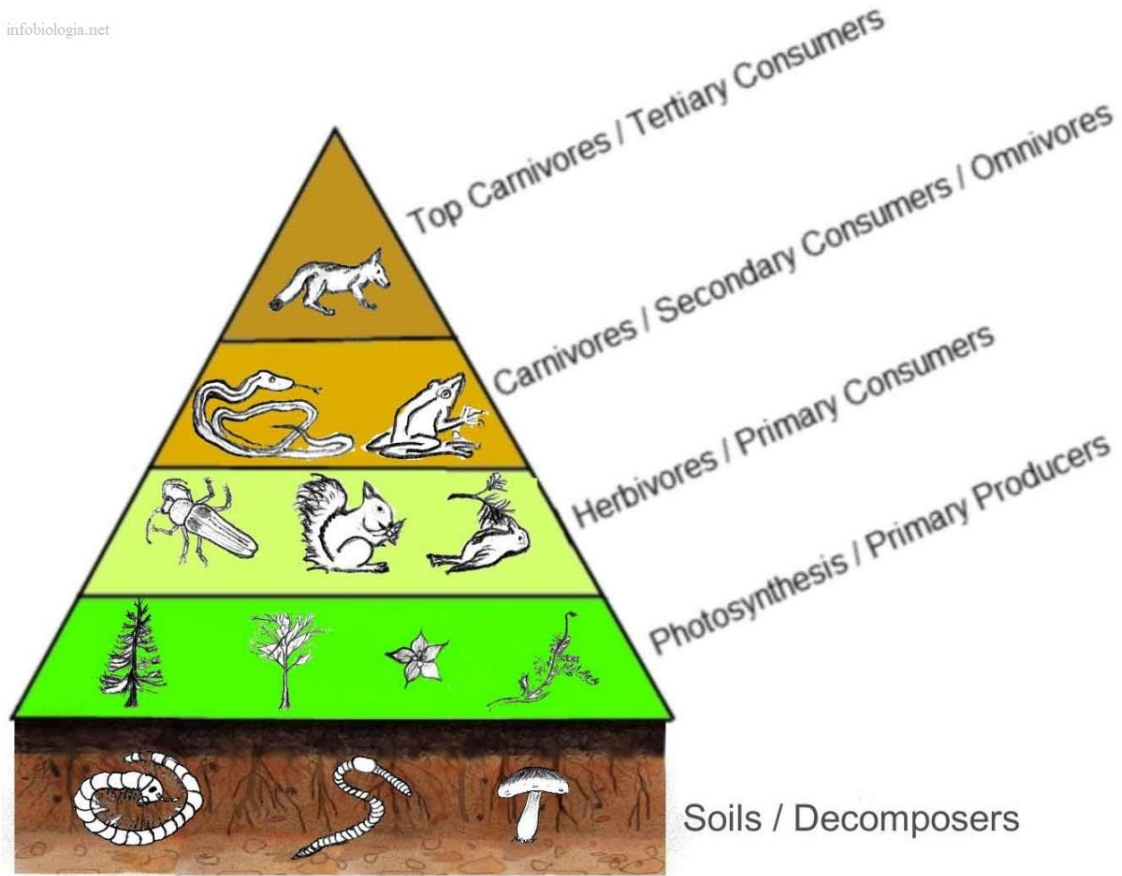


$$x_{n+1} = \rho r x_n \left(1 - \frac{x_n}{k}\right)$$

Heagy et al. (1994), Toniolo et al. (2002), Metta et al. (2010)

# Three-species model

infobiologia.net



Secondary consumers

Primary consumers

Primary producers

# Hastings-Powell's model

Primary producers

$$\frac{dx}{dt} = x(1-x) - \frac{a_1 x}{1+b_1 x} y$$

Primary consumers

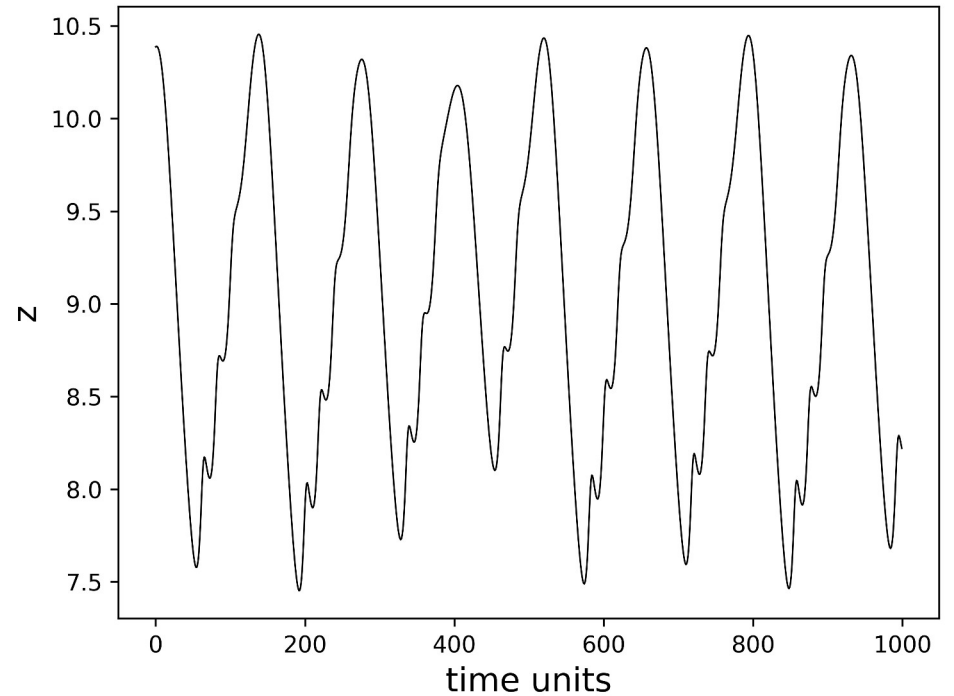
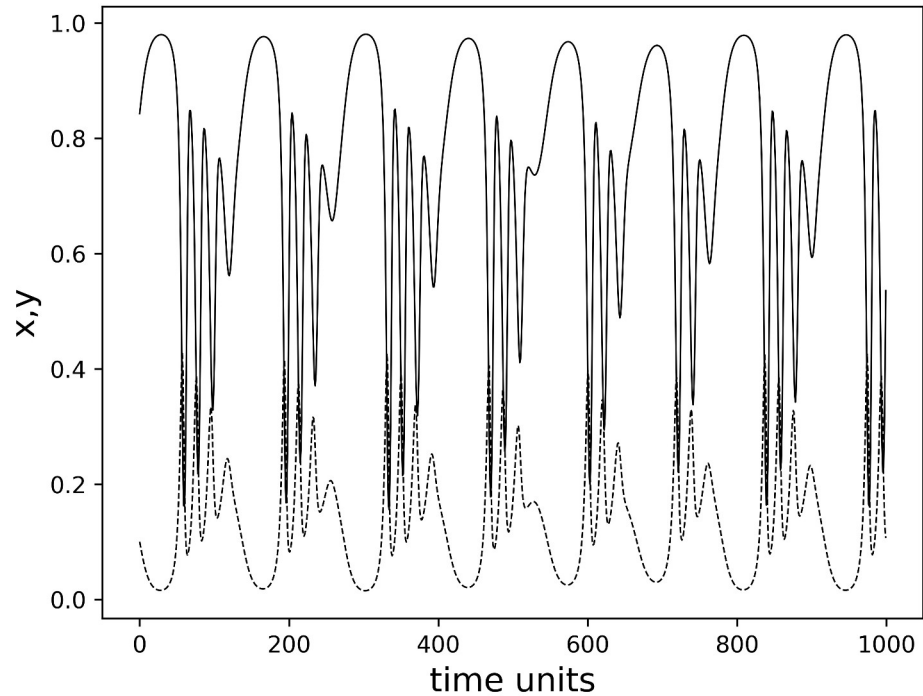
$$\frac{dy}{dt} = \frac{a_1 x}{1+b_1 x} y - \frac{a_2 y}{1+b_2 y} z - d_1 y$$

Secondary consumers

$$\frac{dz}{dt} = \frac{a_2 y}{1+b_2 y} z - d_2 z$$

Vissio Gabriele, Provenzale Antonello (2021), *On-off intermittency in a three-species food chain*, *Mathematics*, 9(14): 1641.

# Hastings-Powell's model



# Stochastic intensity of grazing

Primary producers

$$\frac{dx}{dt} = x(1-x) - \frac{a_1 x}{1+b_1 x} y$$

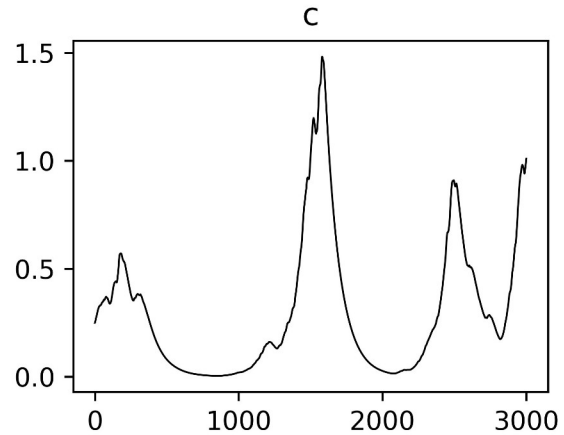
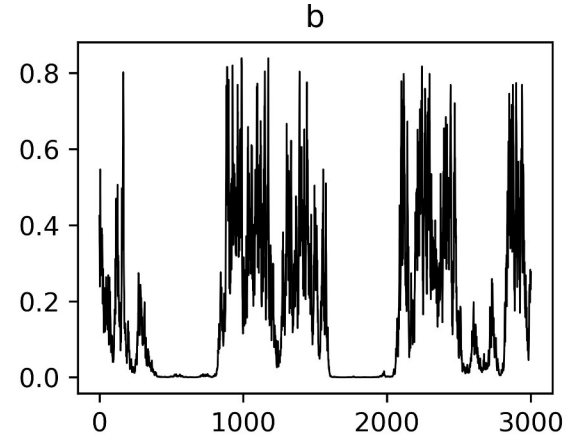
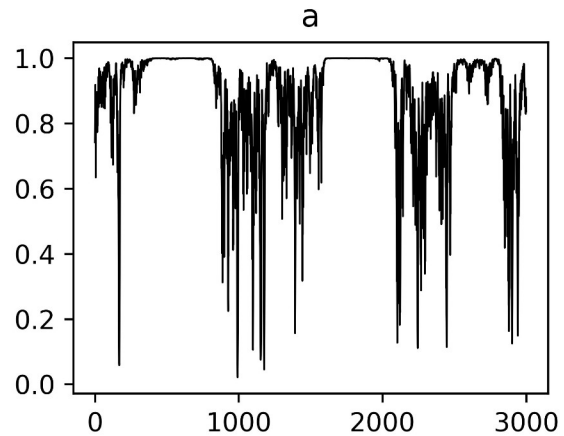
Primary consumers

$$\frac{dy}{dt} = \frac{a_1 x}{1+b_1 x} y - \frac{a_2 y}{1+b_2 y} z - d_1 y$$

Secondary consumers

$$\frac{dz}{dt} = \frac{a_2 y}{1+b_2 y} z - d_2 z$$

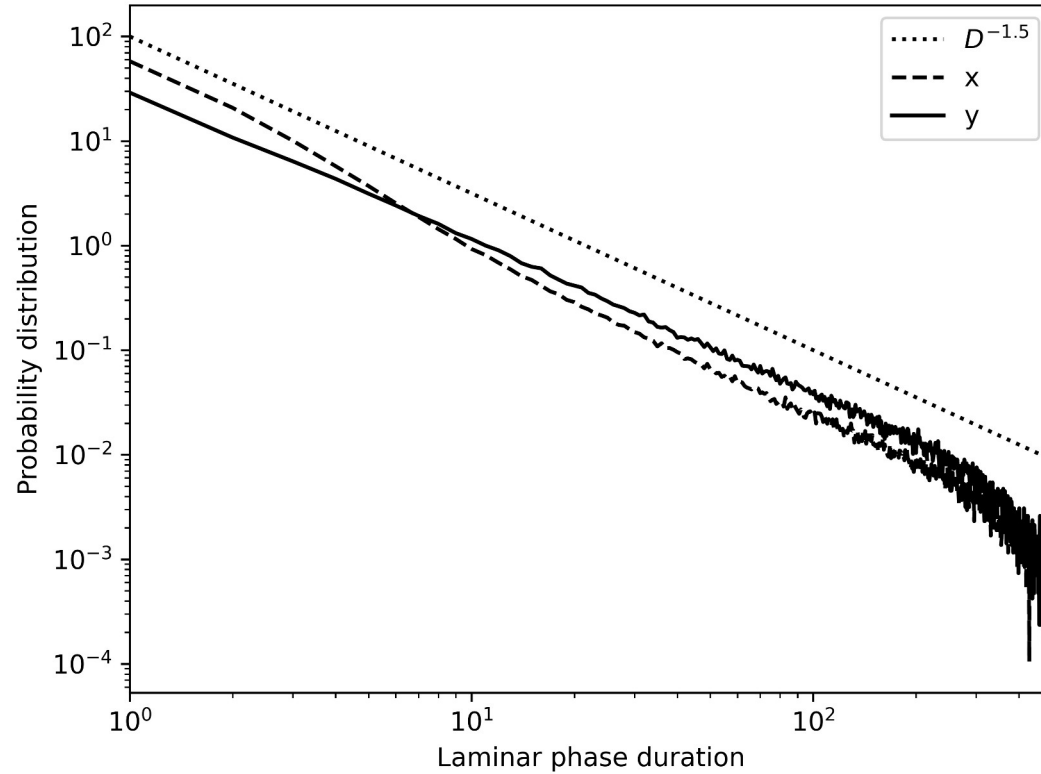
# Stochastic intensity of grazing



$$0 \leq \rho \leq 0.7$$



# Stochastic intensity of grazing



# On-off intermittency

Alternation between laminar phases and sudden bursts

Duration of laminar phases follows a power law

It could explain population outbreaks

# Stochastic carrying capacity

Primary producers

$$\frac{dx}{dt} = x(1-x) - \frac{a_1 x}{1+b_1 x} y$$

Primary consumers

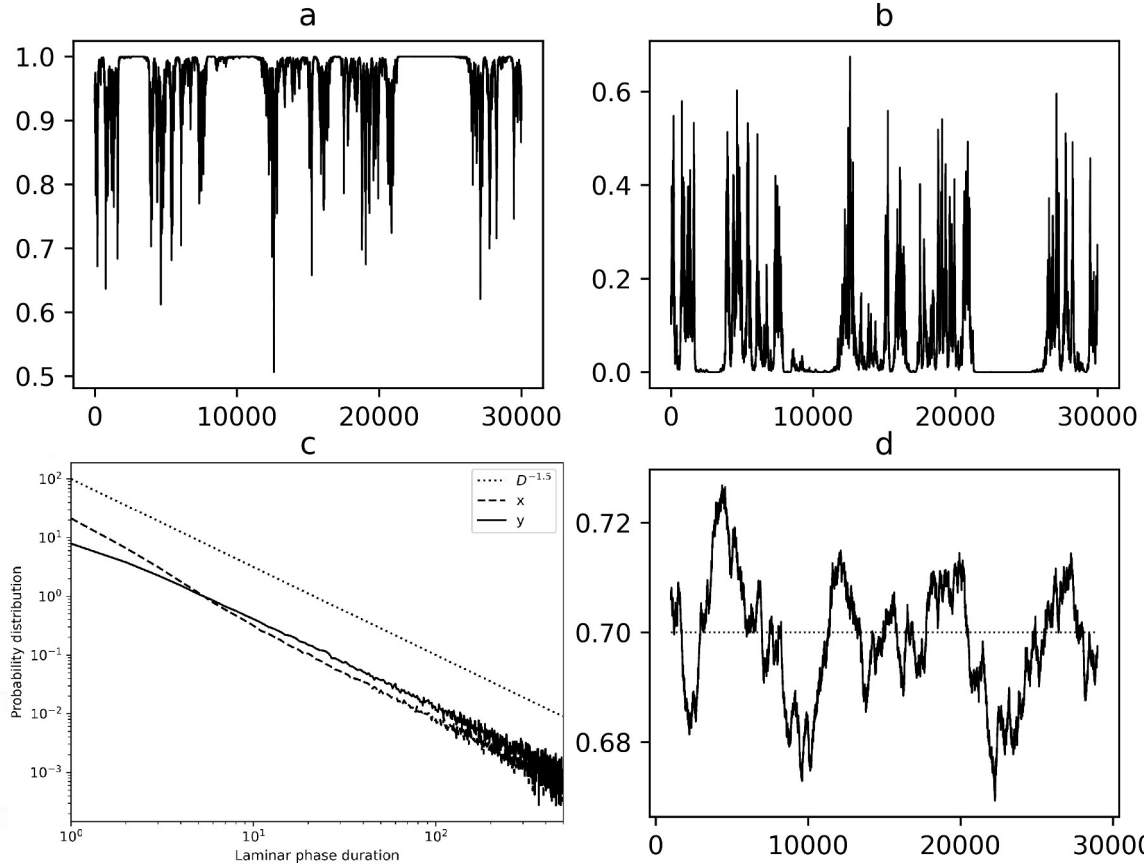
$$\frac{dy}{dt} = \frac{a_1 x}{1+b_1 x} y - \frac{a_2 y}{1+b_2 y} z - d_1 y$$

Secondary consumers

$$\frac{dz}{dt} = \frac{a_2 y}{1+b_2 y} z - d_2 z$$

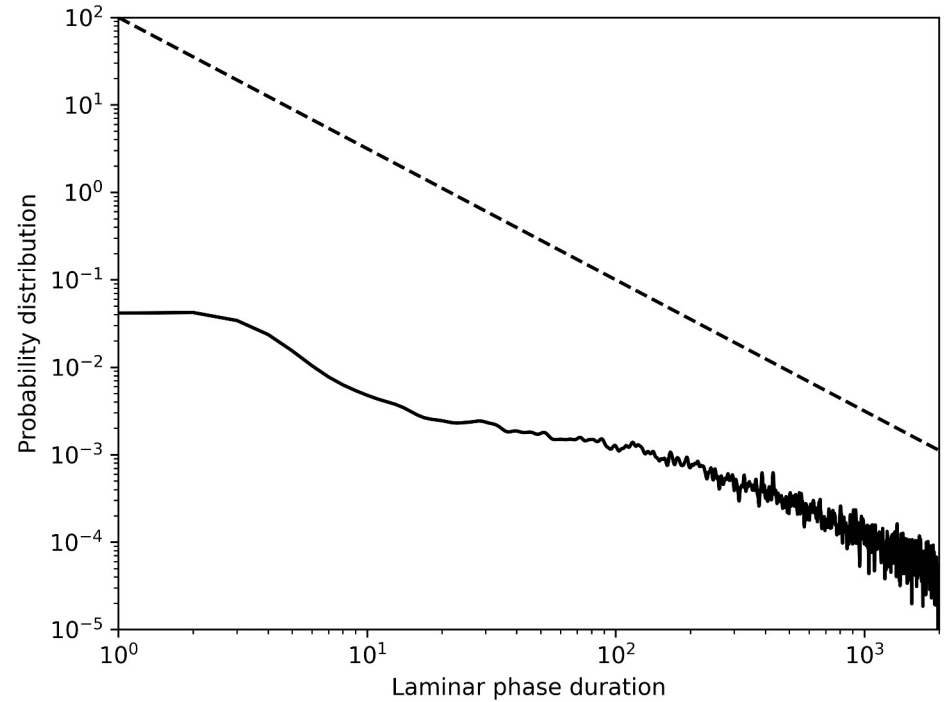
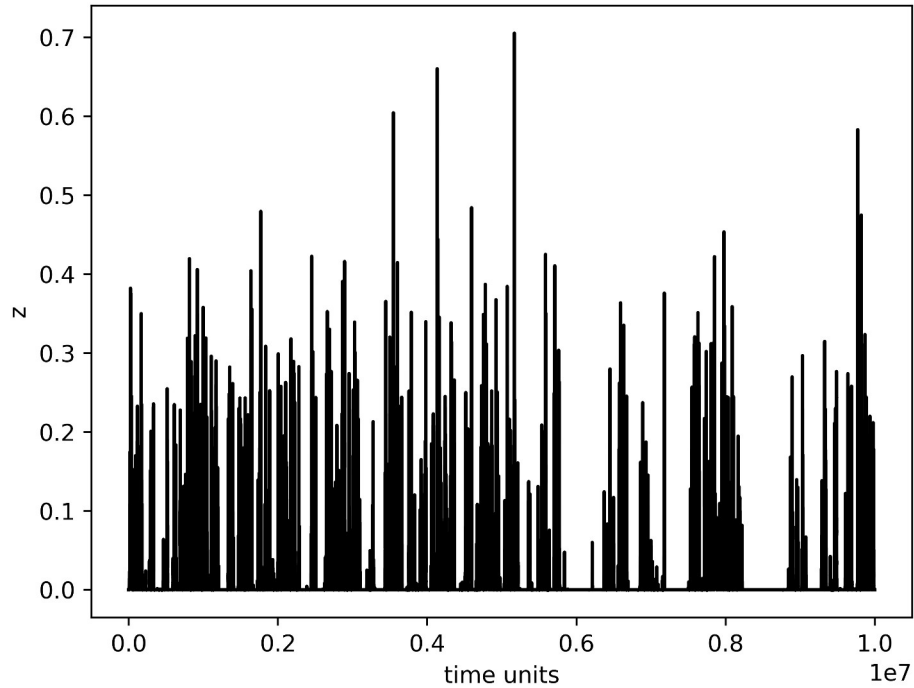
# Stochastic carrying capacity

$$0 \leq \rho \leq 1.4$$



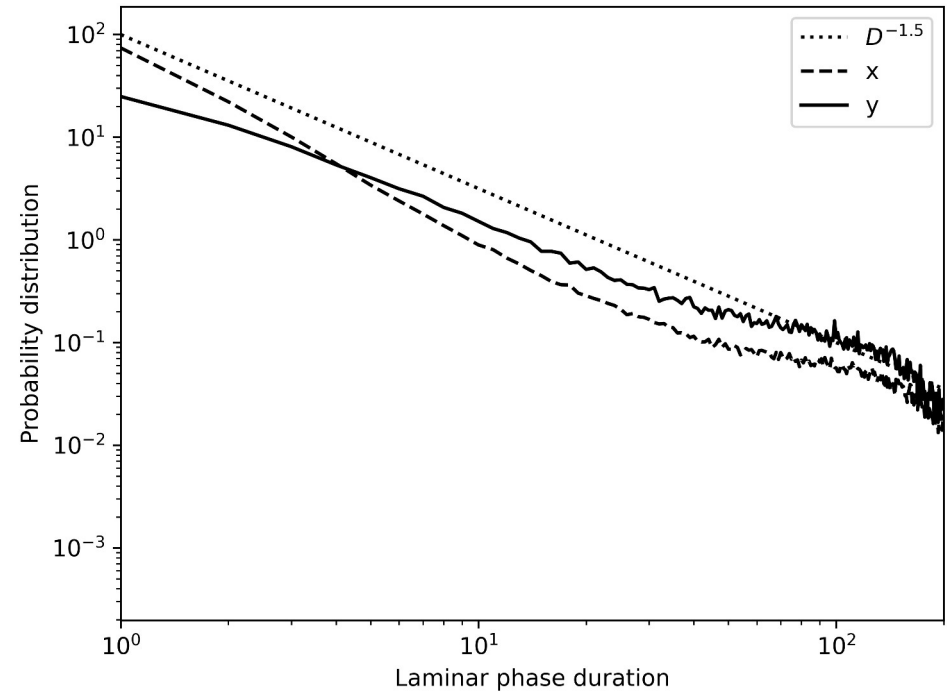
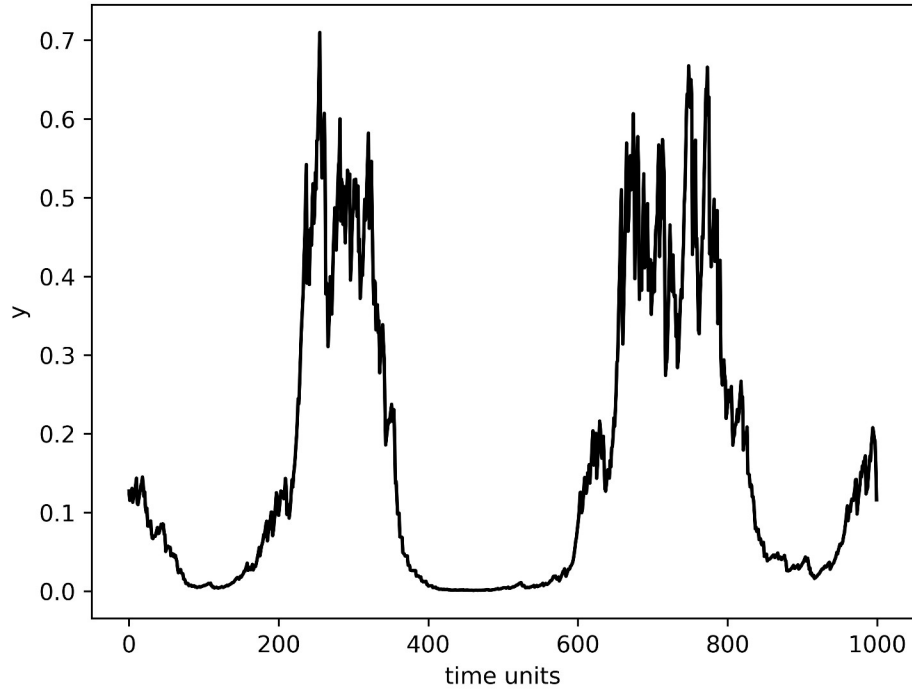
# Stochastic carrying capacity

$$0 \leq \rho \leq 1.5$$



# Stochastic carrying capacity

$$0 \leq \rho \leq 2.1$$



# Beddington's model

Host 
$$N_{t+1} = \delta N_t \exp \left[ r \left( 1 - \frac{N_t}{K} \right) - a P_t \right]$$

Parasitoid 
$$P_{t+1} = b N_t \left[ 1 - \exp(-a P_t) \right]$$

Vissio Gabriele, Provenzale Antonello (2022): *On-off intermittency and irruptions in host-parasitoid dynamics*, Journal of Theoretical Biology 546, 111174

# Beddington's model

Host  $N_{t+1} = \delta N_t \exp \left[ r \left( 1 - \frac{N_t}{K} \right) - a P_t \right]$

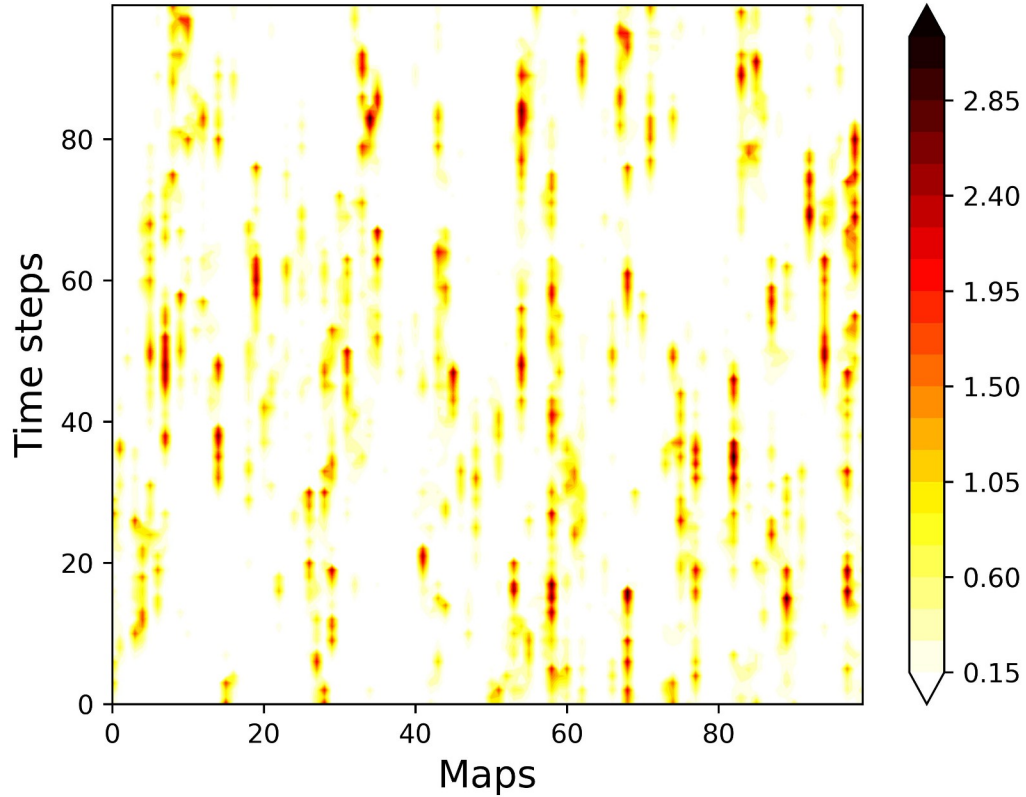
Parasitoid  $P_{t+1} = b N_t \left[ 1 - \exp(-a P_t) \right]$

Local coupling

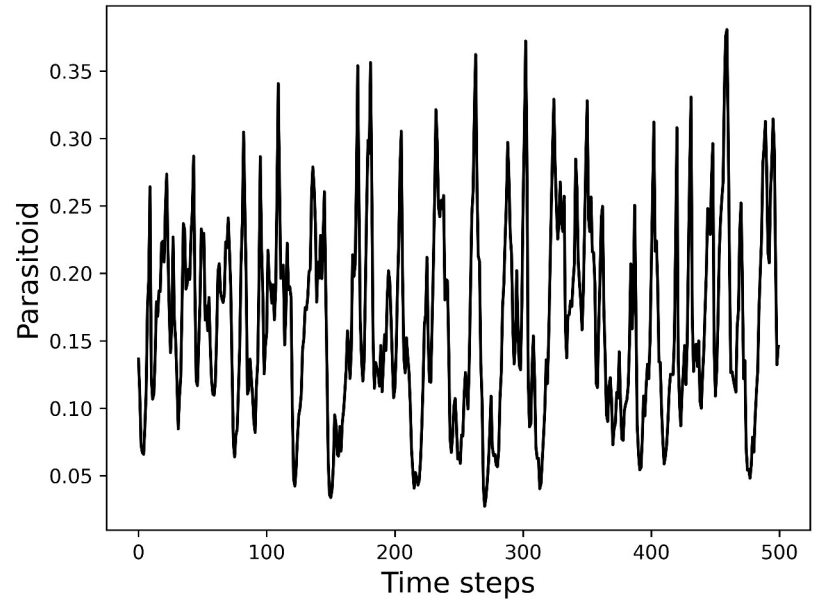
$$N_{t+1}^k = (1 - \gamma) f_N(N_t^k; a_t^k) + \frac{\gamma}{2} \left[ f_N(N_t^{k-1}; a_t^{k-1}) + f_N(N_t^{k+1}; a_t^{k+1}) \right]$$



# Beddington's model



$$0 \leq \delta \leq 0.15, \gamma = 0.001$$



# Beddington's model

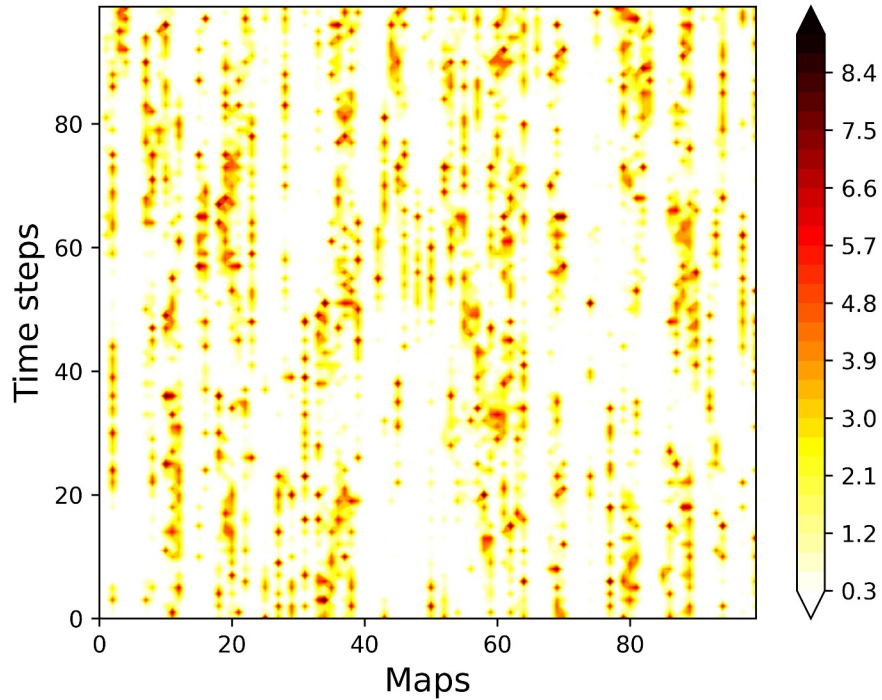
Host 
$$N_{t+1} = \delta N_t \exp \left[ r \left( 1 - \frac{N_t}{K} \right) - a P_t \right]$$

Parasitoid 
$$P_{t+1} = b N_t \left[ 1 - \exp(-a P_t) \right]$$

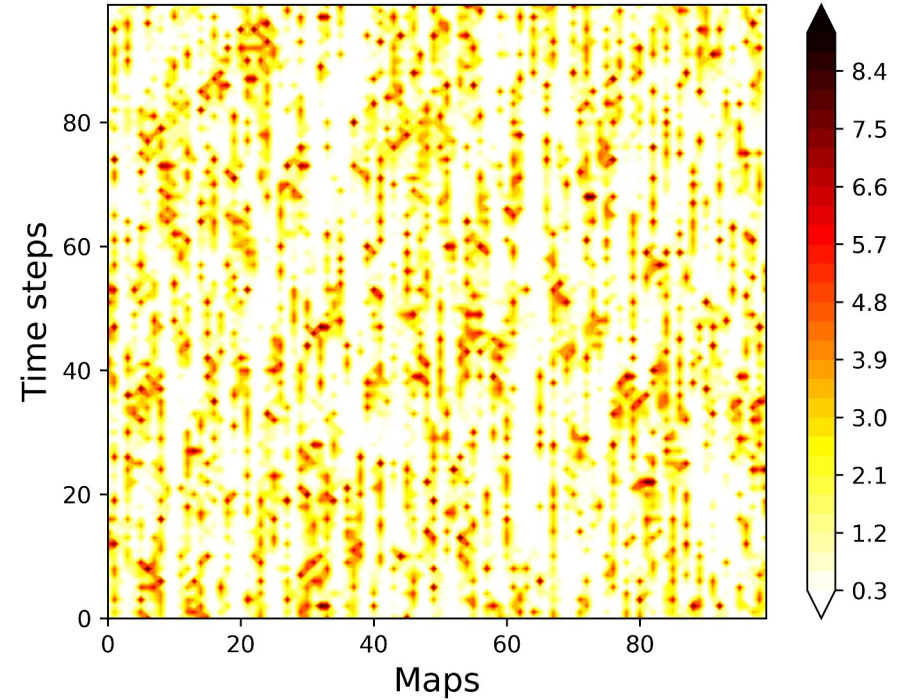
- Local coupling **deletes** on-off intermittency
- Local coupling **produces** on-off intermittency

# Beddington's model

$$0 \leq a \leq 0.6, \gamma = 0$$

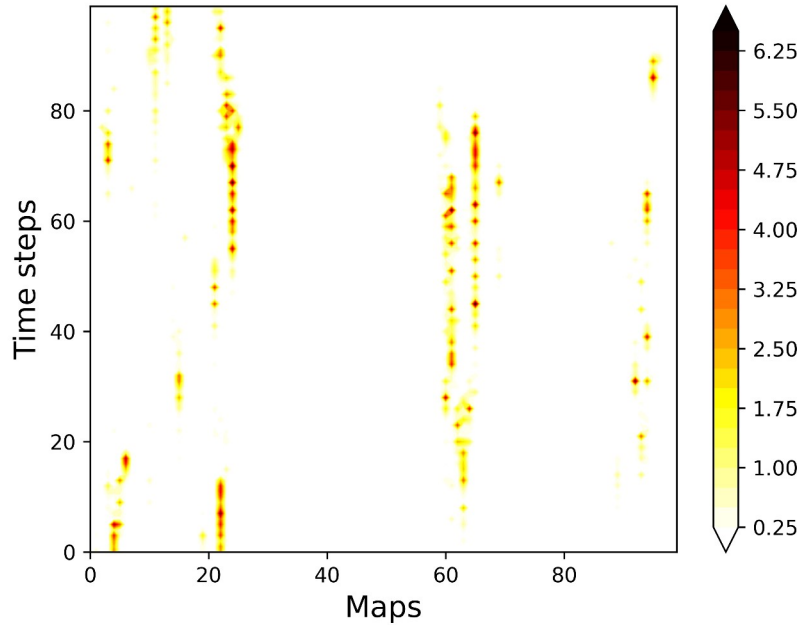


$$0 \leq a \leq 0.6, \gamma = 0.0001$$

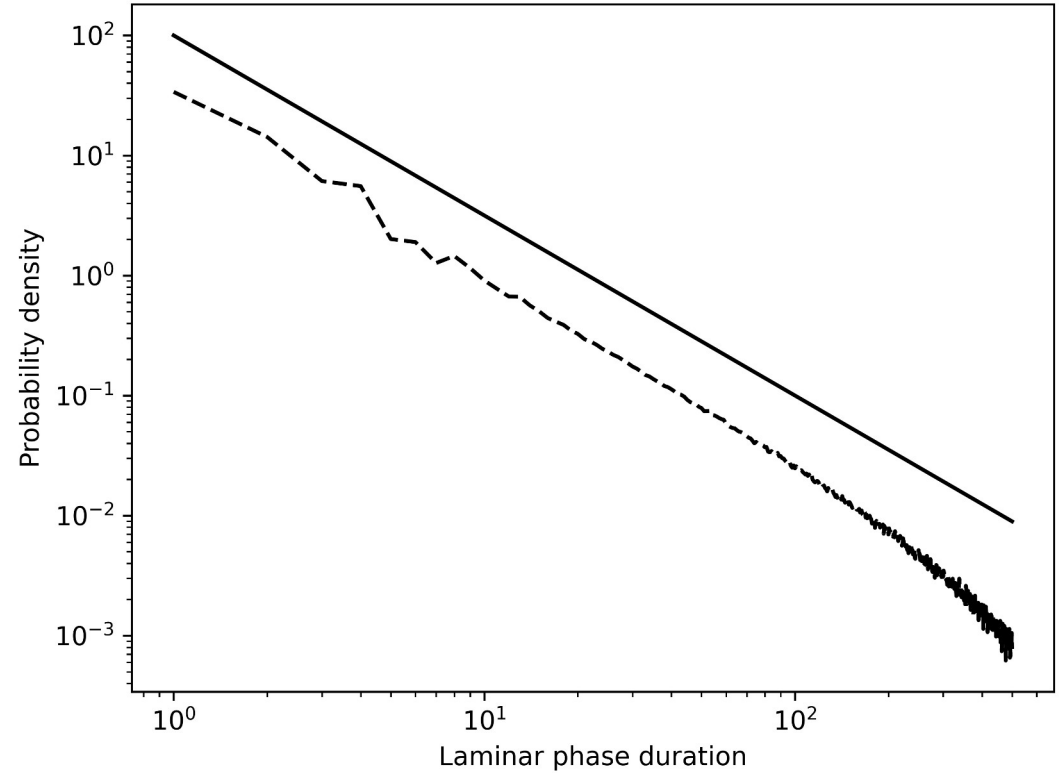


# Beddington's model

$$0 \leq a \leq 0.4, \gamma = 0.005$$



$$PDF \sim D^{-1.5}$$



# Recap

- Environmental forcing can induce on-off intermittency in the system, qualitatively explaining outbreaks in population size
- Intermittency in population not directly involved by the forcing
- Spatial coupling induces or suppresses on-off intermittency

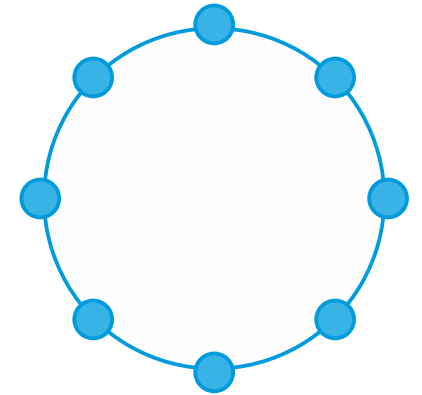
# Forcing with a conceptual model

$$\frac{dp}{dt} = \alpha r p \left( 1 - \frac{p}{k} \right) - \frac{p^2}{1+p^2}$$



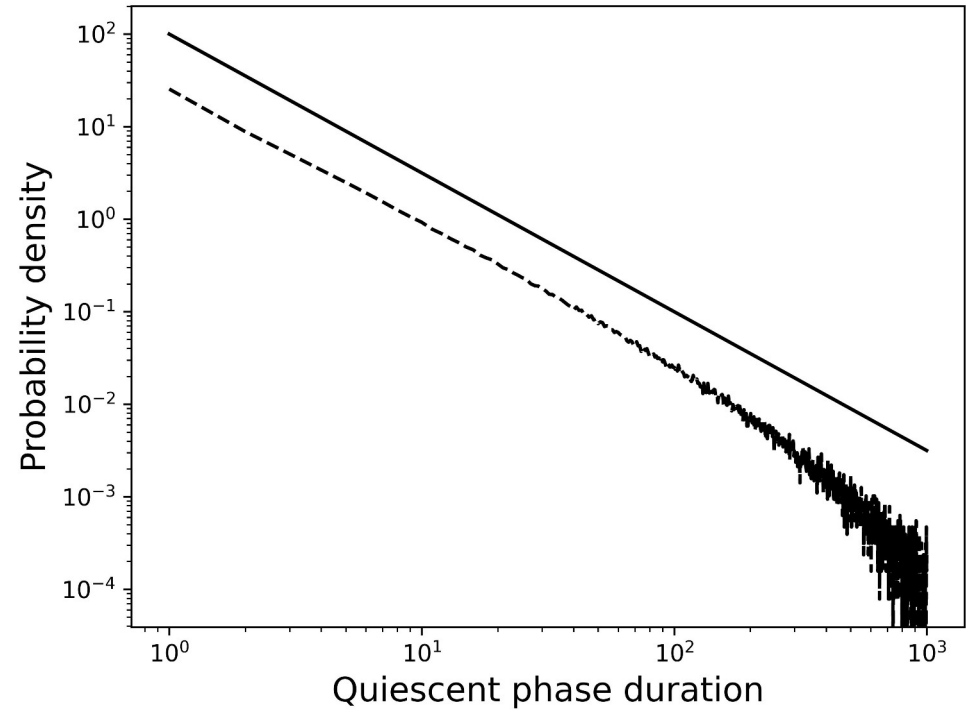
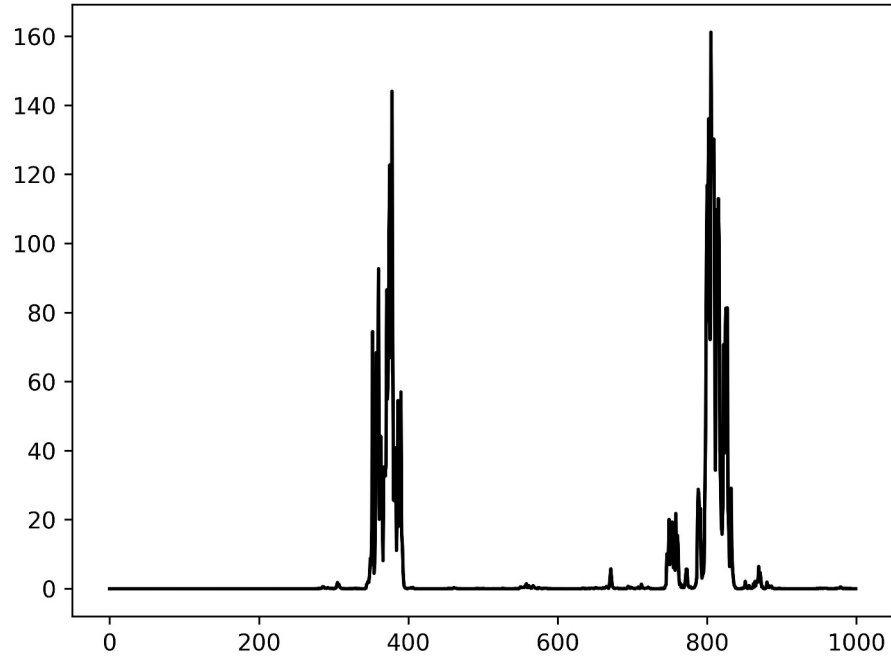
$$\frac{dX_k}{dt} = X_{k-1} (X_{k+1} - X_{k-2}) - \alpha \theta_k - \gamma X_k + F$$

$$\frac{d\theta_k}{dt} = X_{k+1} \theta_{k+2} - X_{k-1} \theta_{k-2} + \alpha X_k - \gamma \theta_k + G$$



Vissio Gabriele, Lucarini Valerio (2020), Mechanics and thermodynamics of a new minimal model of the atmosphere, European Physical Journal Plus 135: 807

# Forcing with a conceptual model



# Environmental Stochasticity and Outbreaks in Population Dynamics

Vissio Gabriele, Provenzale Antonello (2021),  
*On-off intermittency in a three-species food chain*,  
Mathematics, 9(14): 1641.

Vissio Gabriele, Provenzale Antonello (2022):  
*On-off intermittency and irruptions in host-parasitoid dynamics*,  
Journal of Theoretical Biology 546, 111174

